

Conservation of Energy

$$\left(\frac{\partial E}{\partial t}\right)_h + \frac{P_x}{\rho_o} \left(\frac{\partial u_x}{\partial h}\right)_t = 0, \quad (2.3)$$

Constitutive Relation

$$G\left(P_x, \frac{\partial P_x}{\partial t}, \epsilon_x, \frac{\partial \epsilon_x}{\partial t}, E\right) = 0. \quad (2.4)$$

The variables used in these equations are defined as:

h = Lagrangian position coordinate

t = time

P_x = stress in x-direction; considered positive in compression

u_x = particle velocity in x-direction

ρ = density

ϵ_x = total strain in x-direction; positive in compression

E = specific internal energy

Unless otherwise specified, the coordinate x is used to specify displacement along the direction of propagation in Eulerian, or laboratory, coordinates.

For future reference, it is convenient to define two phase velocities as

$$c_u = \left(\frac{\partial h}{\partial t}\right)_{u_x}, \quad (2.5)$$

and

$$c_p = \left(\frac{\partial h}{\partial t}\right)_{P_x}.$$

In terms of these velocities, Fowles²⁵ shows that Eqs. (2.1) - (2.3) can be written as

$$d\rho = \frac{\rho^2}{\rho_o c_u} du_x, \quad (2.6)$$

$$dP_x = \rho_o c_p du_x, \quad (2.7)$$

$$dE = \frac{P_x}{\rho_o c_u} du_x, \quad (2.8)$$

for a mass element (constant h).

In general, the two phase velocities associated with a wave will be functions of stress and will differ in values. However, for steady waves, adiabatic waves, or discontinuous shock fronts, Fowles²⁵ shows that they are both equal. If the elastic precursor is assumed to be a discontinuity in the shock variables, both are equal to the precursor velocity, U . In this case, Eqs. (2.6) - (2.8) can be written in integrated form.⁶ Two of these, which will be discussed often, are

$$P_x = \rho_0 U u_x, \quad (2.9a)$$

and

$$\rho_0 U = \rho(U - u_x). \quad (2.9b)$$

For what follows, it will sometimes be necessary to use Eulerian coordinates directly. For flow in the x -direction, the space and time derivatives are related by⁶

$$\left(\frac{\partial}{\partial t}\right)_h = \left(\frac{\partial}{\partial t}\right)_x + u_x \left(\frac{\partial}{\partial x}\right)_t,$$

$$\left(\frac{\partial}{\partial h}\right)_t = \frac{\rho_0}{\rho} \left(\frac{\partial}{\partial x}\right)_t.$$

The time derivative in Lagrangian coordinates is also referred to as the convective derivative, $\frac{d}{dt}$.

2.2. Constitutive Equation for a Rate-Dependent Material

A constitutive relation for wave propagation in anisotropic media will be developed in this section. Johnson, Jones, and Michaels¹⁴ have treated the general problem of wave propagation in anisotropic media. Their technique will be specialized to the case of longitudinal wave propagation along a $\langle 100 \rangle$ axis in the LiF structure. In the derivation, it will be assumed that (1) the